Behavioural
COMPUTATIONAL SCIENCE LAB

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A Visual Attention-Based Discrete Choice Model to Reveal the Intra-individual Preference Heterogeneity

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Background

Types of preference heterogeneity

Multiple replicates

A choice situation

Multiple individuals

Example

travel mode choice:

Travel time

Travel cost

Comfort level

1st time: I'm in a hurry today, so **travel time** matters most. I'll choose the fastest option.

2nd time: ***Comfort*** matters most to me now. I'll choose the least crowded option.

…

intra-individual heterogeneity

I have a busy schedule and can afford to pay more, so ***travel time*** matters most to me.

I don't mind a moderately long journey, as long as it's ***comfortable***.

…

inter-individual heterogeneity

Reference:
Hess and Rose (2009), Krueger et al. (2021), Kim, S. H. (2023)

Background

DCMs with the preference heterogeneity

Multiple replicates → **A choice situation** ← Multiple individuals

intra-individual heterogeneity

Additive random noise (η_{ns}) on **constant** *personal tastes* μ_{n}.

$$\beta_{ns} = \mu_{\mathrm{n}} + \eta_{\mathrm{ns}},\ \eta_{\mathrm{ns}} \sim \mathrm{N}(0, \Sigma_{\mathrm{n}});$$

Better in-sample fit, but not out-of-sample prediction, than $\eta_{\mathrm{ns}} = 0$.

DCMs in the literature:

- mixed logit models(MXL);
- latent class latent variable models (LCLV);

For indiv n, option j on scenario s, , $U_{njs} = \beta_{ns} X_{njs} + \epsilon_{njs}$
where:
ϵ_{njs}: unobserved utility noise;
η_{ns}: random noise on personal tastes;
μ_{n} is personal tastes.

inter-individual heterogeneity

various *personal tastes* μ_{n} divergent from population mean.

Socioeconomic, demographic, and attitudinal data help identify individual preferences.

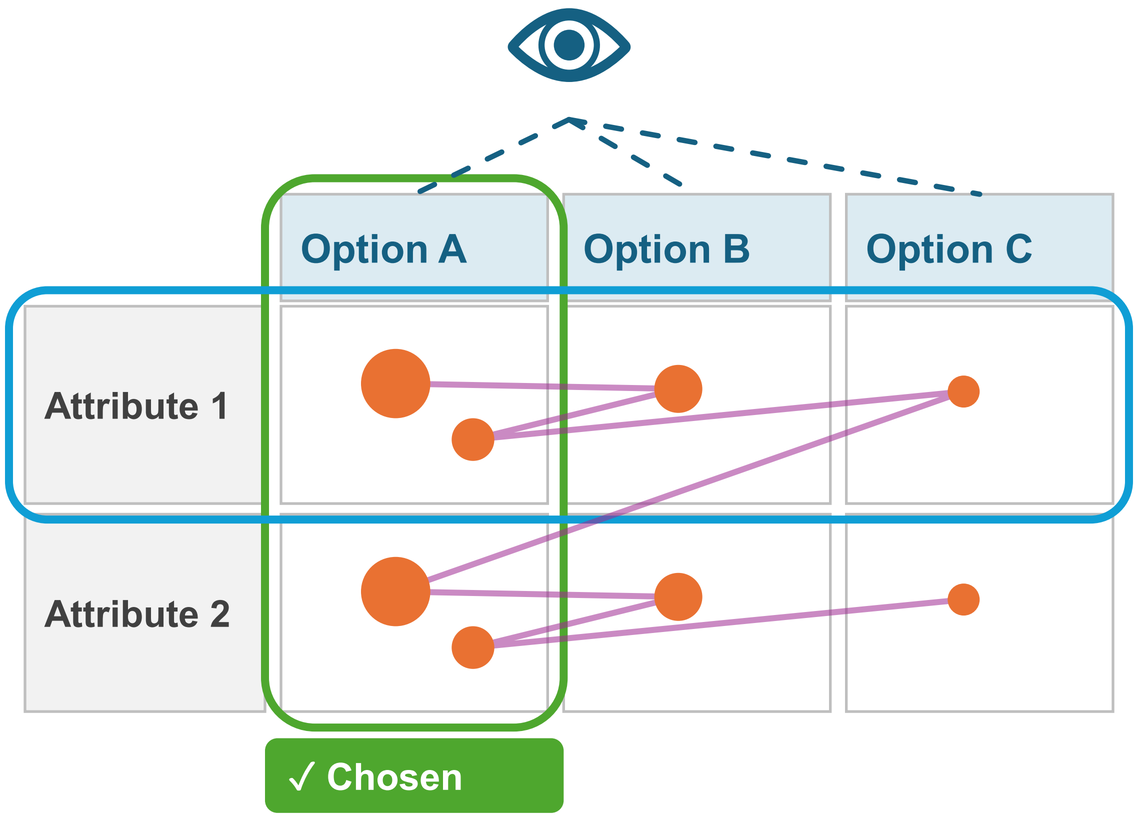
Reference:
Hess and Rose (2009) , Krueger et al. (2021), Kim, S. H. (2023)

Background

Behavioral Study Findings (within Subject)

attention-choice link at <u>alternative level</u>:

The **visual attention weight** on an option <u>positively correlates</u> to its chosen probability.



Dot size ≈ visual attention (gaze fixation). More attention on an option → higher chosen probability.

attention-choice link at <u>attribute level</u>:

The **pairwise comparison** within an attribute related to an option <u>positively correlates</u> its chosen probability.

Reference:
<u>Shimojo et al. (2003)</u>, Noguchi and Stewart (2018).

Background

Research Gap & Motivation

Research Gap

- **Noise, not signal**: within-subject fluctuations enter random utility models as **additive noise**: limited insight, only **modest out-of-sample gains** (Krueger et al., 2021).

- **Case-specific & opaque**: existing intra-individual approaches considering context factors don't **generalize** or explain the **underlying decision process** (Song et al., 2024, Łukawska et al., 2025).

Motivation

- **Process data reflects underlying mechanism: response times (RT)** and **eye-tracking (ET)** give structured signals of preference formation and cognition. Attention strongly explains choice errors within individuals (Khan & Dhar, 2025);

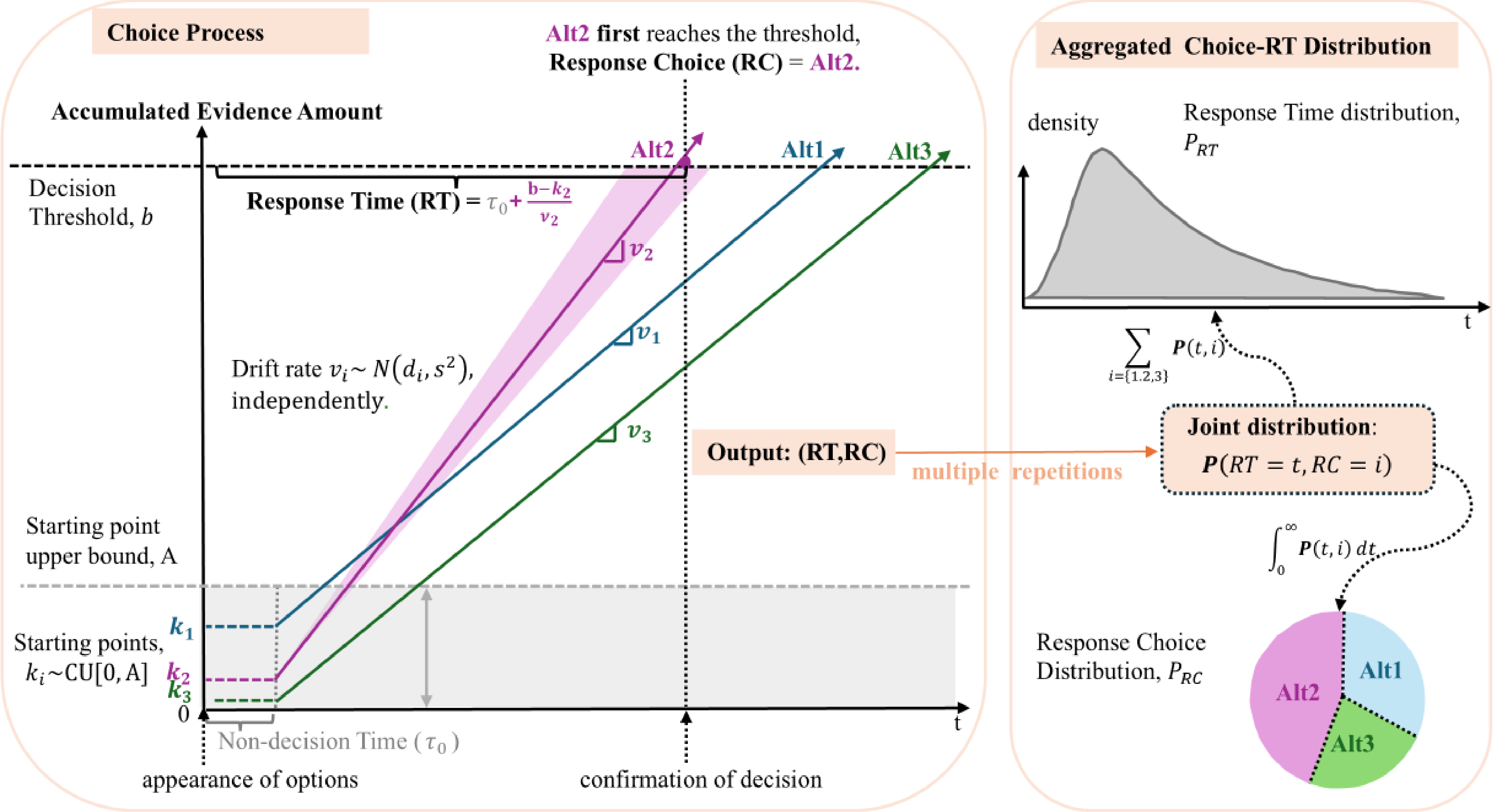
- **Within subject attention–choice link:** stable across choice context and task settings (Shimojo et al., 2003, Noguchi and Stewart ,2018).

An attention-based DCM integrates **ET and RT** to capture the short-term perturbations from stable preferences.

Methodology

Canonical Multi-attribute Ballistic Accumulator (MLBA)

Back-end of MLBA



Reference: Trueblood et al. (2014) and Li and Bansal (2024).

Methodology

Canonical Multi-attribute Ballistic Accumulator (MLBA)

Back-end of MLBA

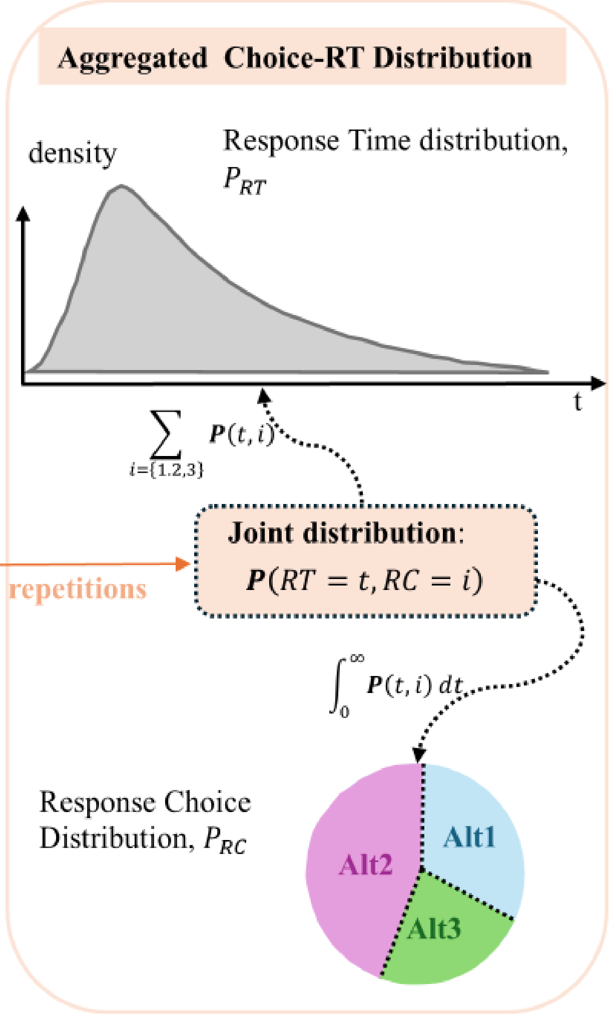
Quick Connection with RUMs

Recall RUM model utility formulation

$$U_{njs} = \beta_{ns} X_{njs} + \epsilon_{njs} \text{ .}$$

- **Accumulated evidence ≈ utility $\boldsymbol{U}$** — just built up over time.
- **Mean drift rate ≈ the observed utility per unit time**, the $\beta_{ns} X_{njs}$ part.
- **Accumulation noise ≈ random error $\boldsymbol{\varepsilon}$ → choice probabilities.**

The **choice-only distribution** of MLBA (under some constraints) ⟺ **Probit**



Reference: Trueblood et al. (2014) and Li and Bansal (2024).

Methodology

Attentional Multi-attribute Ballistic Accumulator (A-MLBA)

Front-end of Attentional-MLBA

Drift rate mean $\boldsymbol{d}_{\mathrm{i}ns}$ for alt i of indiv n for situation s:

$$\boldsymbol{d}_{\mathrm{i}ns} = \boldsymbol{\zeta}_{in} + \underbrace{\sum_{k=1}^{K} \boldsymbol{\beta}_{kn}\left(\boldsymbol{X}_{ikns}\, \boldsymbol{\delta}_{ikns} + \overline{\boldsymbol{X}_{ikn,1:s-1}}(1-\boldsymbol{\delta}_{ikns})\right)}_{\text{isolate goal-driven evaluation}}$$

$$\underbrace{+\boldsymbol{\alpha}_{\mathrm{n}} \sum_{j\in\mathcal{C}, j\neq i} \sum_{k=1}^{K} \boldsymbol{\omega}_{ijkns} \boldsymbol{\lambda}_{\mathrm{n}}\, \boldsymbol{\beta}_{kn}\left(\boldsymbol{X}_{ikns} - \boldsymbol{X}_{jkns}\right),}_{\text{stimuli-driven context effect}}$$

Where intra-attribute comparison weight calibrator is

$$\boldsymbol{\omega}_{ijkns} = \text{standardized}\{\, \boldsymbol{w}_{ikns} \boldsymbol{w}_{jkns}\},$$

$\omega_{ijkns} = 0$: equal attention relative to other attribute pairs

$\omega_{ijkns} > 0$: greater attention

$\omega_{ijkns} < 0$: less attention.

And AOI attendance indicator

$$\boldsymbol{\delta}_{ikns} = \mathrm{I}(\boldsymbol{X}_{ikns} \text{ is fixed}).$$

Other notions:

$\boldsymbol{X}_{ikns}$: the value of attribute k for alt i on situation s.

$\overline{\boldsymbol{X}_{ikn,1:s-1}}$: the average value of attribute k for past 1:s-1 situations.

Personal taste parameters of individual n

$\boldsymbol{\zeta}_{in}$: alternative-specific constant.

$\boldsymbol{\beta}_{kn}$: attribute-specific coefficient.

$\boldsymbol{\lambda}_{1n}$ ($\boldsymbol{\lambda}_{2n}$): sensitivity parameter for gain (loss).

$\boldsymbol{\alpha}_{\mathrm{n}}$: decision strategy weight on context effect.

Stable across trials

Trial-level ET data

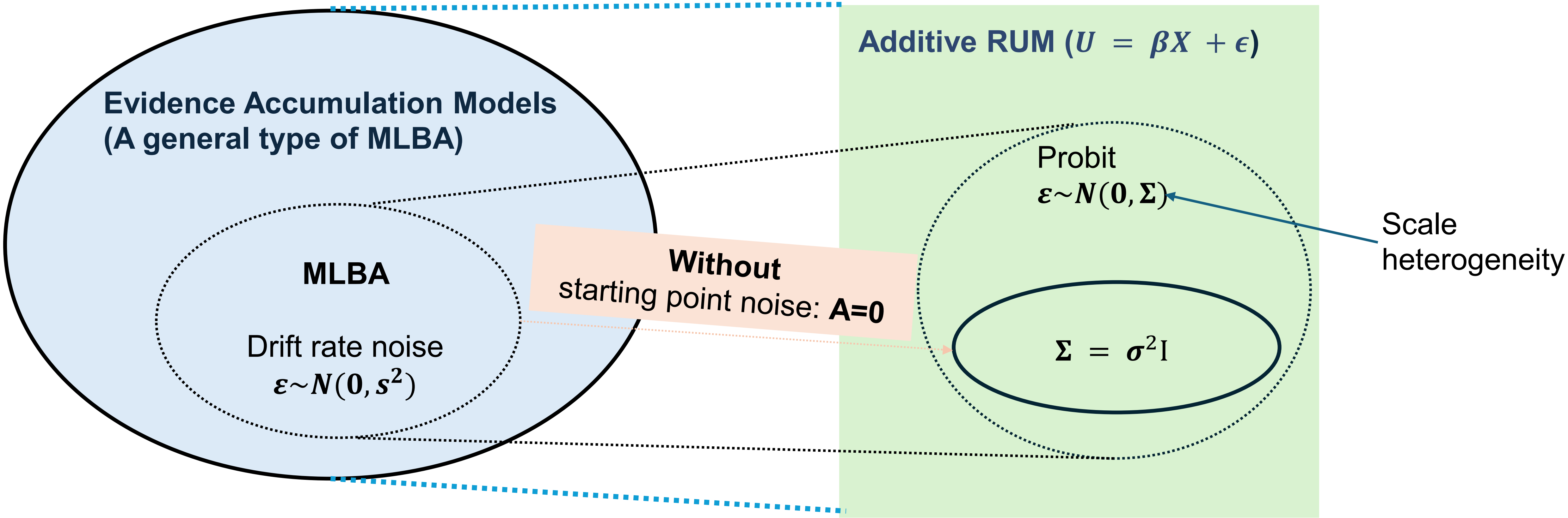
noisy yet informative,

$\boldsymbol{w}_{ikns}$: **fixation duration(count) proportion on AOI** of attribute k for alt i.

Theoretical Interpretation

Why choose MLBA?

If only use Choice-only Distribution of AMLBA:



Not mentioning the benefit of RT brings to RT-involved distribution in terms of parameter estimation and preference recovery.

Reference: Webb (2019) and Li and Bansal (2024).

Theoretical Interpretation
How visual attention explains the preference perturbation?

Recall that

Drift rate mean $\boldsymbol{d}_{ins}$ for alt i of indiv n for situation s:

$$\boldsymbol{d}_{\text{ins}} = \boldsymbol{\zeta}_{\text{in}} + \sum_{k=1}^{K} \boldsymbol{\beta}_{kn}\left(\boldsymbol{X}_{ikns}\,\boldsymbol{\delta}_{ikns} + \overline{\boldsymbol{X}_{ikn,1:s-1}}(1-\boldsymbol{\delta}_{ikns})\right)$$

$$+\boldsymbol{\alpha}_{\text{n}} \sum_{j\in\mathcal{C},j\neq i} \sum_{k=1}^{K} \boldsymbol{\omega}_{ijkns}\boldsymbol{\lambda}_{\text{n}}\,\boldsymbol{\beta}_{kn}\left(\boldsymbol{X}_{ikns} - \boldsymbol{X}_{jkns}\right),$$

Where intra-attribute comparison weight calibrator is

$$\boldsymbol{\omega}_{ijkns} = \text{standardized}\{\boldsymbol{w}_{ikns}\boldsymbol{w}_{jkns}\},$$

$\omega_{ijkns} = 0$: equal attention relative to other attribute pairs

$\omega_{ijkns} > 0$: greater attention

$\omega_{ijkns} < 0$: less attention.

And AOI attendance indicator

$$\boldsymbol{\delta}_{ikns} = \mathrm{I}(\boldsymbol{X}_{ikns}\text{ is fixed}).$$

Special case 1

When all AOIs are being viewed and paid equal attention:

$$\boldsymbol{\delta}_{ikns} = 1, \boldsymbol{\omega}_{ijkns} = 0, \forall \mathrm{i, j, k, n, s}$$

Theoretical Interpretation
How visual attention explains the preference perturbation?

Drift rate mean $\boldsymbol{d}_{\boldsymbol{i}\mathrm{ns}}$ for alt i of indiv n for situation s:

$\boldsymbol{d}_{\boldsymbol{i}\mathrm{ns}} = \boldsymbol{\zeta}_{\boldsymbol{i}\mathrm{n}} + \sum_{\boldsymbol{k}=\mathbf{1}}^{\boldsymbol{K}} \boldsymbol{\beta}_{\boldsymbol{k}\mathrm{n}} \boldsymbol{X}_{\boldsymbol{ik}\mathrm{ns}}$

is equivalent to

The observed utility in additive utility model

$\mathrm{V}_{\boldsymbol{i}\mathrm{ns}} = \boldsymbol{\zeta}_{\boldsymbol{i}\mathrm{n}} + \sum_{\boldsymbol{k}=\mathbf{1}}^{\boldsymbol{K}} \boldsymbol{\beta}_{\boldsymbol{k}\mathrm{n}} \boldsymbol{X}_{\boldsymbol{ik}\mathrm{ns}},$

where **only inter-individual heterogeneity** is considered.

$$\beta_{ns} = \beta_{\mathrm{n}} + \eta_{\mathrm{ns}}, \qquad \eta_{\mathrm{ns}} = 0$$

Special case 1

When all AOIs are being viewed and paid equal attention:

$$\boldsymbol{\delta}_{\boldsymbol{ik}\mathrm{ns}} = 1, \boldsymbol{\omega}_{\boldsymbol{ijk}\mathrm{ns}} = 0, \forall \mathrm{i, j, k, n, s}$$

Choice-only A-MLBA = Mix-probit model with inter-individual heterogeneity

Theoretical Interpretation

How visual attention explains the preference perturbation?

Recall that

Drift rate mean $\boldsymbol{d}_{i\mathrm{ns}}$ for alt i of indiv n for situation s:

$$\boldsymbol{d}_{i\mathrm{ns}} = \boldsymbol{\zeta}_{i\mathrm{n}} + \sum_{\boldsymbol{k}=\mathbf{1}}^{\boldsymbol{K}} \boldsymbol{\beta}_{k\mathrm{n}}\left(\boldsymbol{X}_{\boldsymbol{ik}\mathrm{ns}}\, \boldsymbol{\delta}_{\boldsymbol{ik}\mathrm{ns}} + \overline{\boldsymbol{X}_{\boldsymbol{ik}\mathrm{n},1:\mathrm{s}-1}}\left(1 - \boldsymbol{\delta}_{\boldsymbol{ik}\mathrm{ns}}\right)\right)$$

$$+\boldsymbol{\alpha}_{\mathrm{n}} \sum_{\boldsymbol{j}\in\mathcal{C},\boldsymbol{j}\neq\boldsymbol{i}} \sum_{\boldsymbol{k}=\mathbf{1}}^{\boldsymbol{K}} \boldsymbol{\omega}_{\boldsymbol{ijk}\mathrm{ns}} \boldsymbol{\lambda}_{\mathrm{n}}\, \boldsymbol{\beta}_{\boldsymbol{k}\mathrm{n}}\left(\boldsymbol{X}_{\boldsymbol{ik}\mathrm{ns}} - \boldsymbol{X}_{\boldsymbol{jk}\mathrm{ns}}\right),$$

Where intra-attribute comparison weight calibrator is

$$\boldsymbol{\omega}_{\boldsymbol{ijkns}} = \mathrm{standardized}\{\, \boldsymbol{w}_{\boldsymbol{ikns}} \boldsymbol{w}_{\boldsymbol{jkns}}\},$$

$\omega_{ijkns} = 0$: equal attention relative to other attribute pairs

$\omega_{ijkns} > 0$: greater attention

$\omega_{ijkns} < 0$: less attention.

And AOI attendance indicator

$$\boldsymbol{\delta}_{\boldsymbol{ik}\mathrm{ns}} = \mathrm{I}(\boldsymbol{X}_{\boldsymbol{ik}\mathrm{ns}} \text{ is fixed}).$$

Special case 2

When all AOIs are being viewed

but

the AOIs within the same attribute gain the equal attention:

$$\boldsymbol{\delta}_{\boldsymbol{ik}\mathrm{ns}} = 1, \boldsymbol{\omega}_{\boldsymbol{ijk}\mathrm{ns}} \sim \mathrm{N}(0, \mathrm{s_{kn}}^2), \forall \mathrm{k}, \mathrm{n}$$

Theoretical Interpretation

How visual attention explains the preference perturbation?

Drift rate mean $\boldsymbol{d}_{i\mathrm{ns}}$ for alt i of indiv n for situation s:

$$\boldsymbol{d}_{i\mathrm{ns}} = \boldsymbol{\zeta}_{i\mathrm{n}} + \sum_{k=1}^{K} \boldsymbol{\beta}_{k\mathrm{n}} \boldsymbol{X}_{ik\mathrm{ns}} + \boldsymbol{\alpha}_{\mathrm{n}} \sum_{j\in\mathcal{C},j\neq i} \sum_{k=1}^{K} \boldsymbol{\omega}_{k\mathrm{ns}} \boldsymbol{\lambda}_{\mathrm{n}}\, \boldsymbol{\beta}_{k\mathrm{n}} \left(\boldsymbol{X}_{ik\mathrm{ns}} - \boldsymbol{X}_{jk\mathrm{ns}}\right)$$

For choice-only AMLBA (A=0), only drift rate difference matters (l as referenced alt)

$$\boldsymbol{d}_{i\mathrm{ns}} - \boldsymbol{d}_{\mathrm{lns}} = \boldsymbol{\zeta}_{i\mathrm{n}} - \boldsymbol{\zeta}_{\mathrm{ln}} + \sum_{k=1}^{K} (\boldsymbol{\beta}_{k\mathrm{n}} + \eta_{\mathrm{nks}}*)(\boldsymbol{X}_{ik\mathrm{ns}} - \boldsymbol{X}_{jk\mathrm{ns}}),$$

with $\boldsymbol{\eta}_{nks}* = 2\boldsymbol{\omega}_{k\mathrm{ns}} \boldsymbol{\lambda}_{\mathrm{n}}\, \boldsymbol{\alpha}_{\mathrm{n}} \sim \mathrm{N}(0, \boldsymbol{\lambda}_{\mathrm{n}}\, \boldsymbol{\alpha}_{\mathrm{n}} \mathrm{s}_{\mathrm{kn}}{}^{2}),$

is equivalent to

The observed utility in additive utility model

$$\mathrm{V}_{i\mathrm{ns}} - \mathrm{V}_{\mathrm{lns}} = \boldsymbol{\zeta}_{i\mathrm{n}} - \boldsymbol{\zeta}_{\mathrm{ln}} + \sum_{k=1}^{K} (\boldsymbol{\beta}_{k\mathrm{n}} + \eta_{\mathrm{nks}})(\boldsymbol{X}_{ik\mathrm{ns}} - \boldsymbol{X}_{jk\mathrm{ns}}),$$

where **intra-individual heterogeneity** is treated as random noise $\boldsymbol{\eta}_{ns} \sim \mathrm{N}(0, \Sigma_{\mathrm{n}})$;

Special case 2

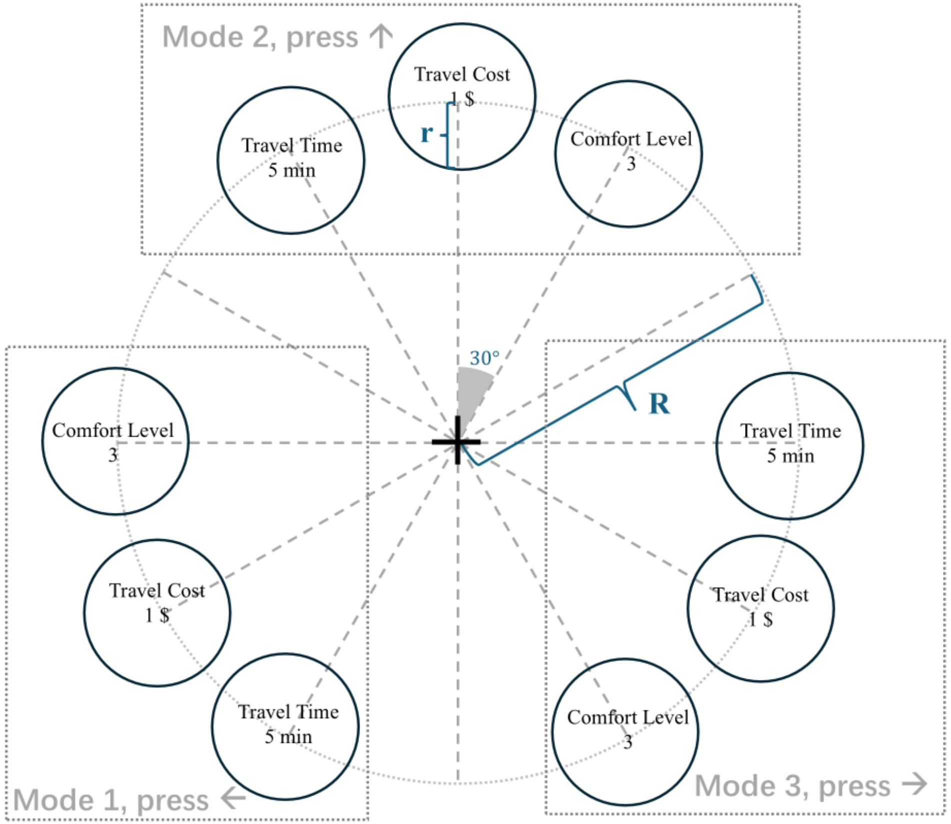
When all AOIs are being viewed
but
the AOIs within the same attribute gain the equal attention:

$$\boldsymbol{\delta}_{ik\mathrm{ns}} = 1, \boldsymbol{\omega}_{ijk\mathrm{ns}} \sim \mathrm{N}(0, \mathrm{s}_{\mathrm{kn}}{}^{2}), \forall \mathrm{k}, \mathrm{n}$$

Choice-only A-MLBA = Mix-probit model with inter- and intra- individual heterogeneity

Empirical Validation:
DCE Layout Design

Please note that all text, (dashed/solid/dotted) lines, area in grey and blue are used for illustration and won't appear in the formal experiment.



Choice Task

- **3** travel-mode alternatives, each with **3** attributes
- ***Travel cost*** (SGD) **·** ***travel time*** (min) · ***comfort*** (1–5 scale)

Sample & Procedure

- **40** participants × **150** trials (**50** unique)
- 50-trial blocks with 2–3 min rest between blocks
- ET and RT recorded on every trial: SR1000 plus

Salience Control

- Attributes kept **equidistant** from a central fixation cross and **randomize** the sequence.
- Reduces **peripheral-vision**, and **fixation-pass-by** noise.

Empirical Validation:
Data Collection Procedure

1 Compulsive fixation location reset (2s)

2 Ternary choice task (free time)

...

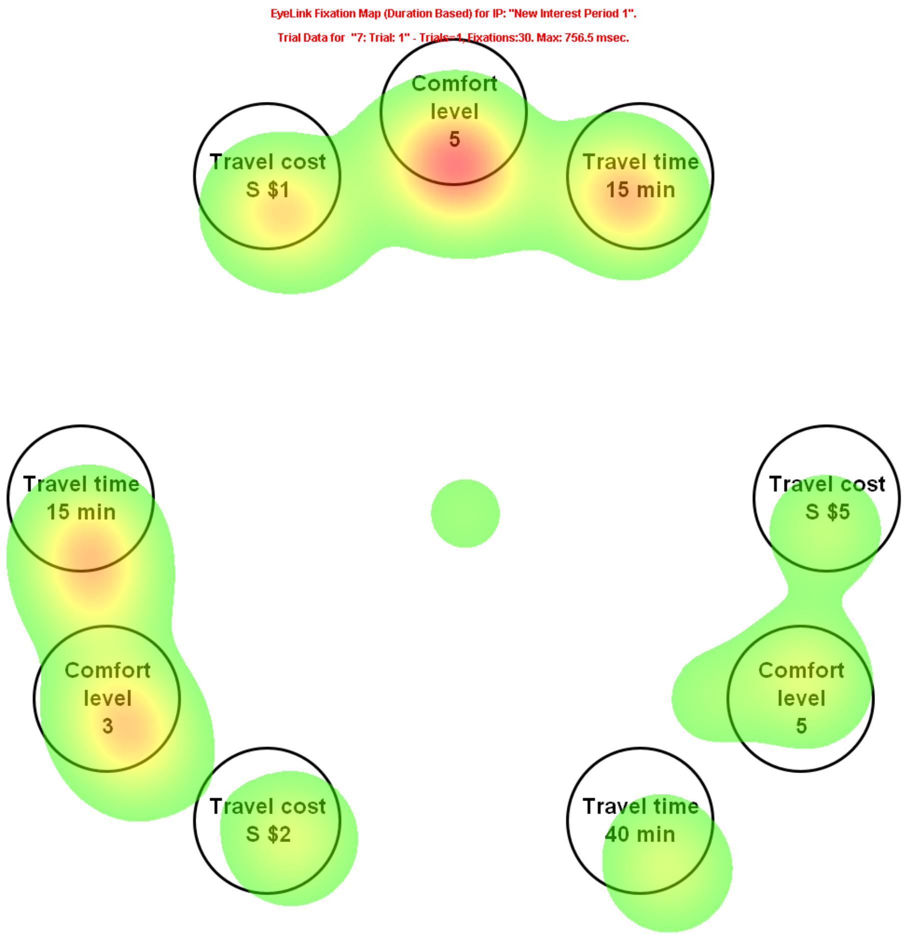
Go through 1 2 for 150 choice situations in total (each individual)

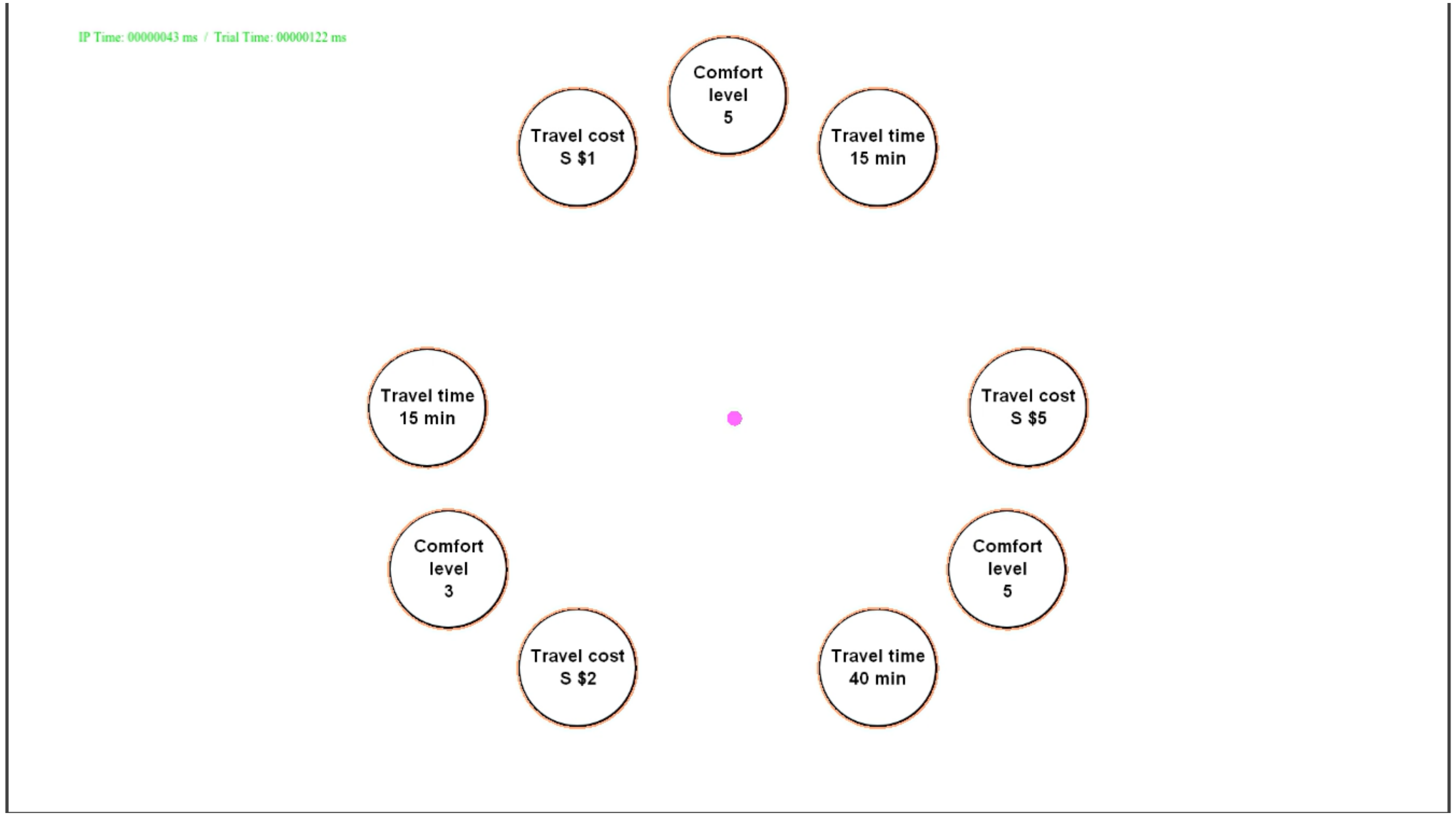
5 blocks with 4 breaks.

Chosen choice, RT and ET are collected.

Empirical Validation:

Trial-level ET data output (example: top option is chosen.)





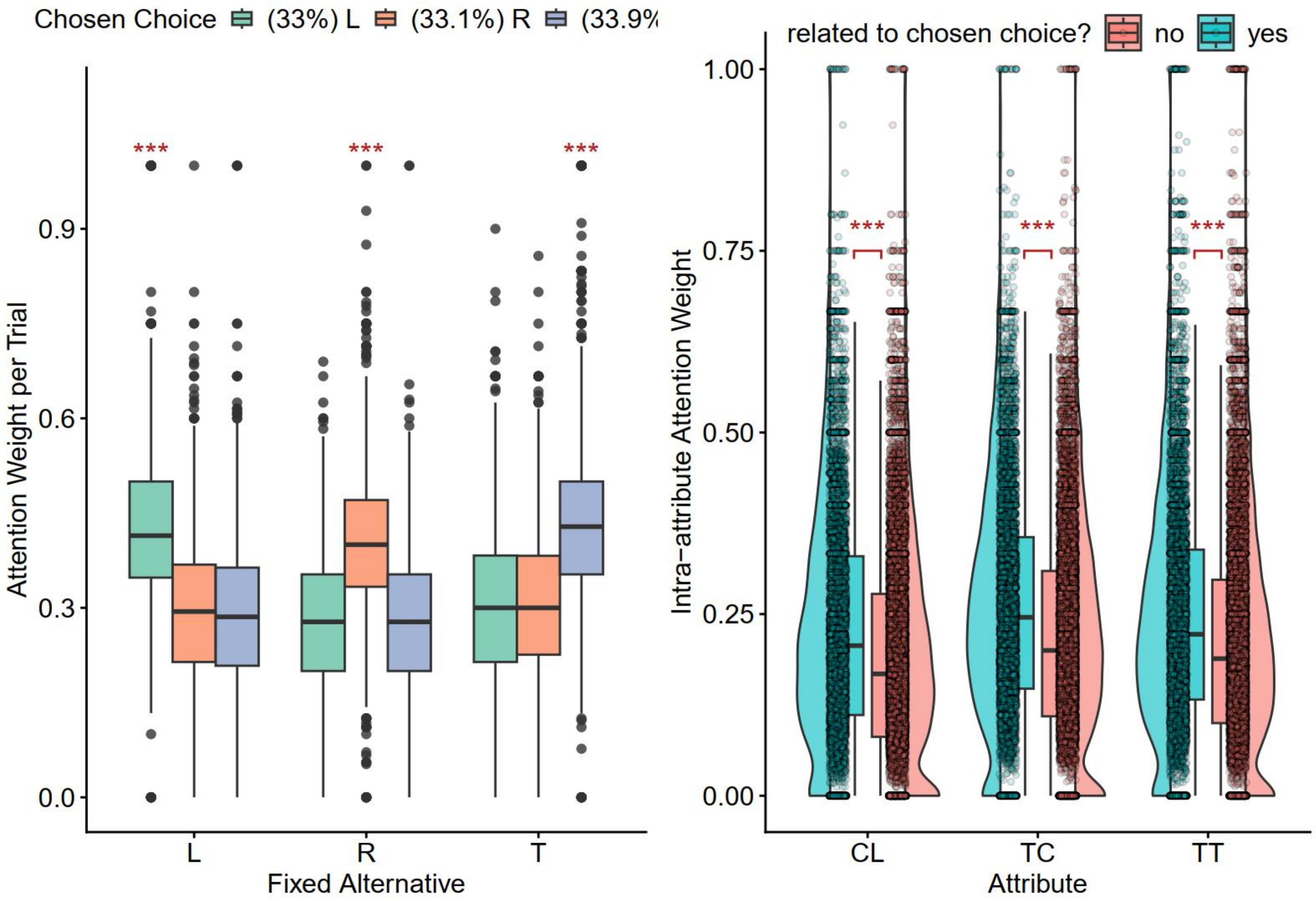
Trial-level fixation count share w_{ikns}.

Unbalanced Intra-attribute comparison weight ω_{kns}

Empirical Validation:
Descriptive Analysis

ET-choice link at <u>alt level</u>:

The **visual attention weight** on an option <u>positively correlates</u> to its chosen probability.



ET-choice link at <u>attribute level</u>:

The **pairwise comparison** within an attribute related to an option <u>positively correlates</u> its chosen probability.

Reference:
<u>Krueger et al. (2021)</u>

Empirical Validation:
Metrics & Benchmark Models

Brier Score

$$\mathrm{BS} = \frac{1}{\mathrm{NS}} \sum_{\mathrm{n,j,s}} (p_{\mathrm{njs}} - y_{\mathrm{njs}})^2$$

lower is better.

Predicted Choice Accuracy

$$\mathrm{Acc} = \frac{1}{\mathrm{NS}} \sum_{\mathrm{n,s}} 1[\mathrm{argmax}_{\mathrm{j}}\, p_{\mathrm{njs}} = y_{\mathrm{ns}}]$$

higher is better.

Models used for prediction check

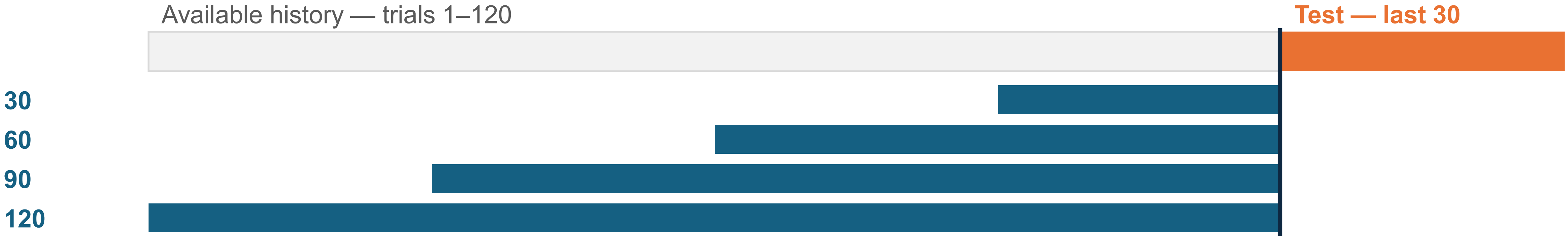
MXL — mixed logit with random coefficients
(inter only; inter- and intra-noise both)

MProbit — mixed Probit with random coefficients
(inter only; inter- and intra-noise both)

V-MLBA — only RT is used;
intra-attention weight is estimated by attribute difference

A-MLBA — attention-augmented, uses ET

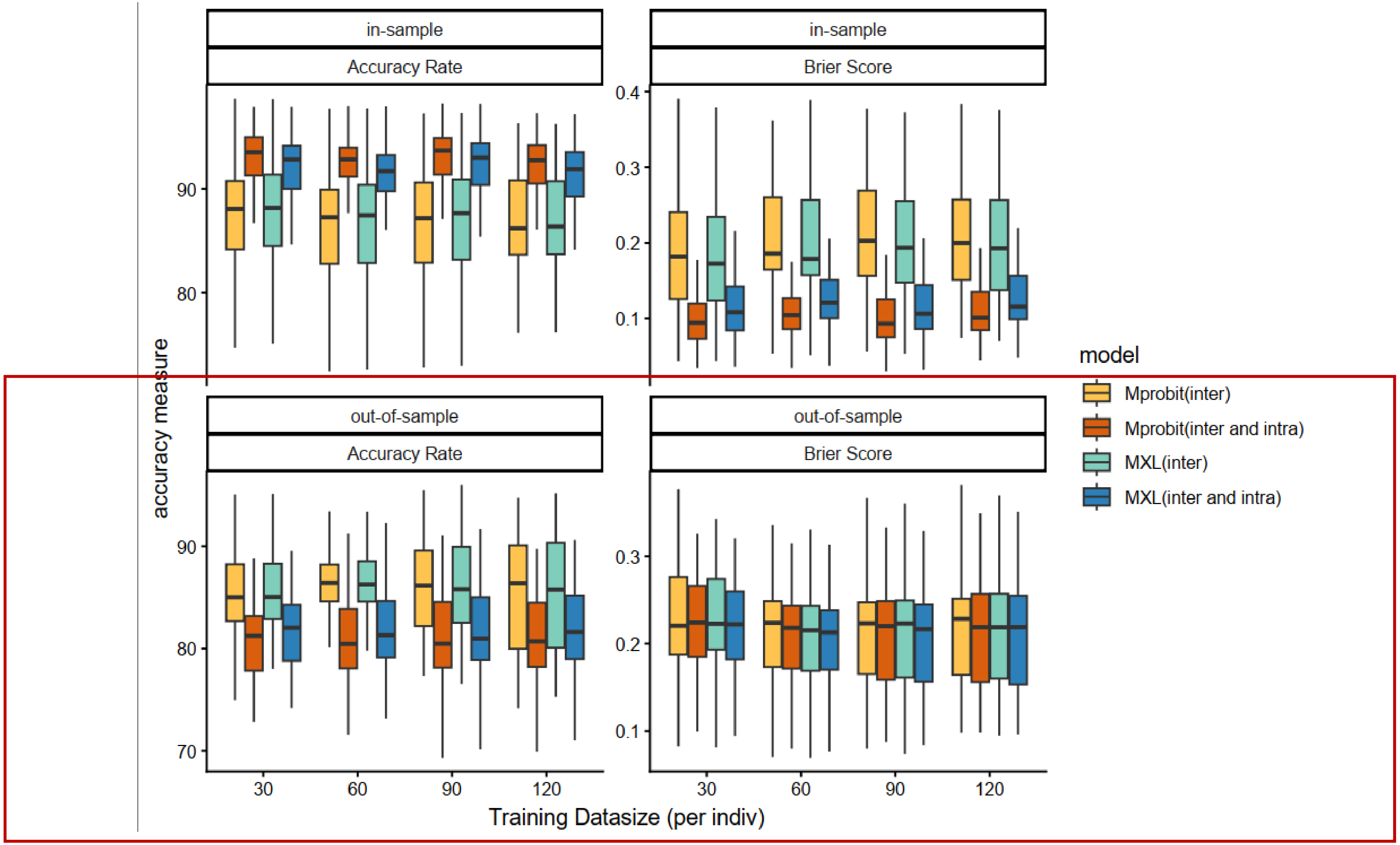
Training / Test Split (per participant · 150 trials in order)



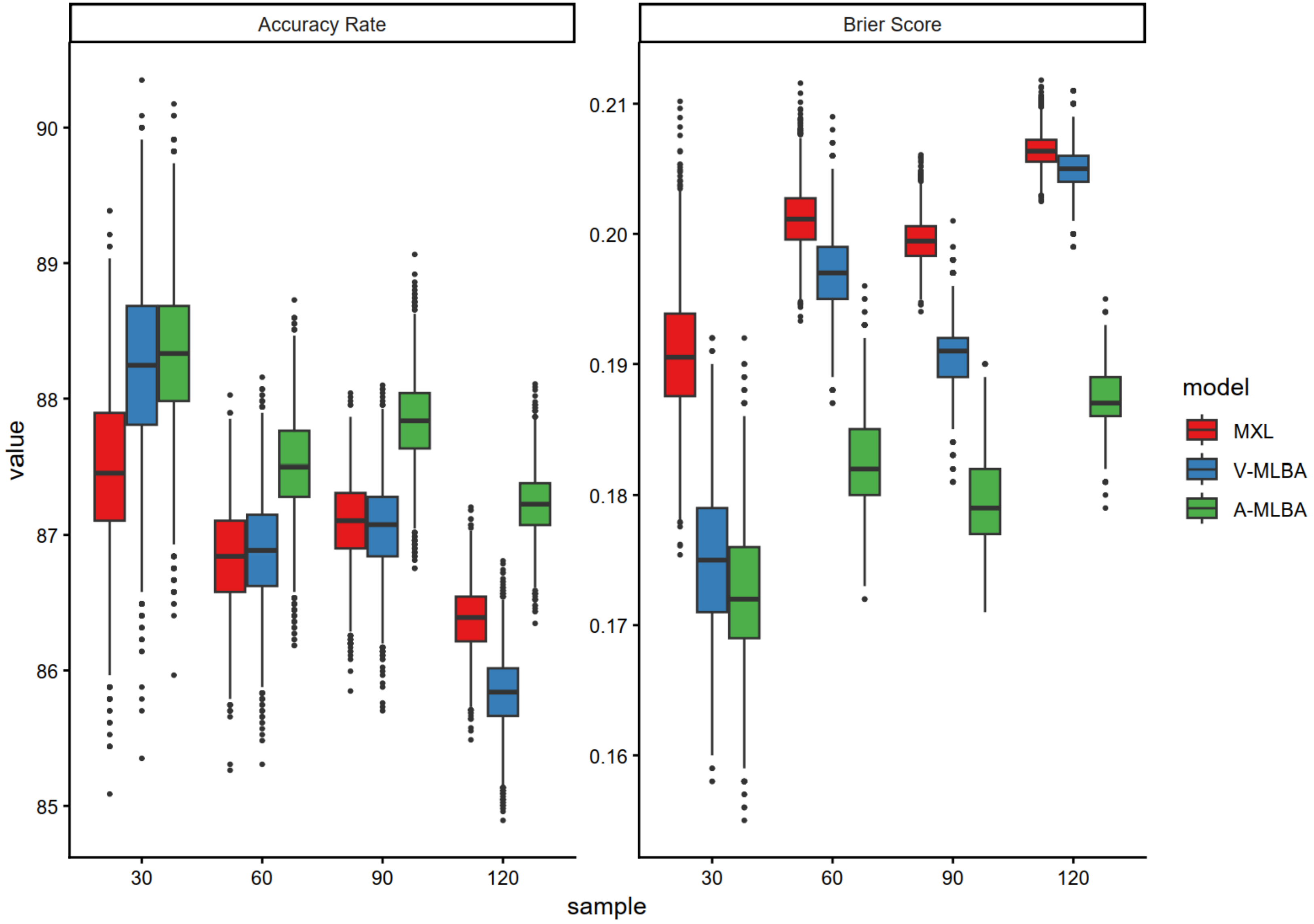
Last 30 trials held out for validation; training uses the preceding 30 / 60 / 90 / 120 trials.

Empirical Validation:

Modest Improvement of Mixed-RUM with additional intra-individual heterogenuity



Empirical Validation:
Out of Sample Model Performance (within-individual analysis)



Conclusion and Discussions

Conclusions

- **Theoretically,** the proposed **A-MLBA** integrating **ET and RT process data** to investigate the choice process mechanism.

- **A-MLBA** type attentional model **nests classic RUMs** as special cases, bridging evidence-accumulation and random-utility models.

- **Empirically**, **intra-individual preference perturbations** isn't always noise, it reveals some signals of decision-making process.

- **A-MLBA** explains **short-term deviations** from stable preferences via attribute- and alternative-level attention–choice links.

Discussions & Future Work

- **Trial-level ET** is informative to **capture the temporal preference perturbation** within subject.

- Generalized attentional DCMs can guide **choice architecture**, **information-display design**, and **personalized recommender systems.**

Thank You!

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